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NONPARAMETRIC TESTS OF INDEPENDENCE FOR CENSORED DATA.(U)  
JUL 80 R M KORWAR, R C DAHIYA F49620-79-C-0105

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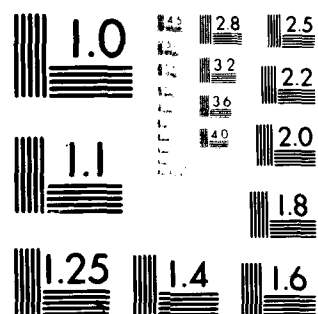
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NONPARAMETRIC TESTS OF INDEPENDENCE  
FOR CENSORED DATA

by

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Ramesh M. Korwar<sup>1</sup> and Ram C. Dahiya<sup>1</sup>

AFOSR Final Technical Report,

July, 1980

The University of Massachusetts  
Department of Mathematics and Statistics  
Amherst, Massachusetts

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<sup>1</sup>Research sponsored by the Air Force Office of Scientific Research, AFSC,  
USAF under Contract F49620-79-C-0105

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# ABSTRACT

The work accomplished is represented by six Tech Reports already issued. Papers based on two of them are accepted for publication and are soon to be published in two of the leading journals in statistics. One other is submitted for publication. And yet another will appear in a Proceedings of a conference on nonparametric statistics.

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## 1. Introduction.

The accomplishments are represented by the following Technical Reports (listed in chronological order) written and issued from time to time:

- [1] Korwar, R.M. (1979). On the uniformly minimum variance unbiased estimators of the variance and its reciprocal of an inverse Gaussian distribution.
- [2] Dahiya, R.C. (1979). Estimating the population sizes of different types of organisms in a plankton sample.
- [3] Dahiya, R.C. (1979). An improved method of estimating an integer parameter by maximum likelihood.
- [4] Dahiya, R.C. (1979). Pearson goodness-of-fit test when the sample size is unknown.
- [5] Korwar, R.M. (1980). Nonparametric estimation of a bivariate survivorship function with doubly censored data.
- [6] Hollander, M., and Korwar, R.M. (1980). Nonparametric Bayesian estimation of the horizontal distance between two populations.

## 2. A Brief Description of the Work by Korwar.

In [5] above, the problem of estimation of a bivariate survivorship function with doubly censored data is considered. A "self-consistent" estimator is developed by first "reducing" the problem to that of estimation in a singly and right censored situation. This estimator is shown to satisfy a likelihood equation, and its uniqueness is investigated. The results obtained naturally parallel those obtained by Campbell (1979) in the singly censored case.

This paper is submitted for publication to the Annals of Statistics. A revision is underway. An abstract of the paper has appeared in the Bulletin of the Institute of Mathematical Statistics (IMS Bulletin 9, #3 (1980), 131, #80t-39).

In [1] above, it is argued that the two-parameter inverse Gaussian distribution has useful applications in a wide variety of fields such as reliability and biometry. The uniformly minimum variance unbiased estimator of the mean of the distribution is known and is the sample mean. However, no such estimator of the variance is reported in the literature. Here the uniformly minimum variance unbiased estimators of the variance and its

reciprocal are derived.

A revised version of the Tech Report is accepted for publication in the Journal of the American Statistical Association and will appear in that journal in September, 1980.

In [6] above, Hollander and Korwar consider the problem of nonparametric Bayesian estimation of the horizontal distance  $\Delta(x) = G^{-1}(F(x)) - x$  between two distribution functions  $F$  and  $G$  at  $x$ . Doksum (1974) has shown  $\Delta(x)$  to be a useful measure of the difference, at each  $x$ , between the populations defined by continuous distribution functions  $F$  and  $G$ . Here the present authors assume that  $G$  is known, and develop a Bayesian nonparametric estimator  $\tilde{\Delta}_n(x)$  of  $\Delta(x)$  based on a random sample of  $n$   $X$ 's from  $F$ . The estimator  $\tilde{\Delta}_n$  is, for weighted squared-error loss, Bayes with respect to Ferguson's (1973) Dirichlet process prior. Using a result of Korwar and Hollander (1976), the Bayes risk of  $\tilde{\Delta}_n$  is evaluated for the case when  $G$  is uniform.

This is an invited paper (Hollander) and will appear in a North-Holland Publication of the Proceedings of the János Bolyai Nonparametric Conference, Budapest, Hungary, June, 1980.

In Section 2 of Korwar and Dahiya (1979) a nonparametric estimator of the bivariate survival function, when only one known variable is assumed to be censored, was proposed and studied. Korwar has now further studied this estimator for its large sample properties. From the properties of the univariate Kaplan-Meier estimator (1958) it follows that the above mentioned Korwar-Dahiya estimator is pointwise consistent. From the work of Campbell and Földes (1980) it also follows that under mild conditions the estimator obeys a law of the iterated logarithms. Details will be reported later on as part of the work under a new AFOSR Grant.

### 3. A Brief Description of the Work by Dahiya.

In [2] above, Dahiya considers a problem related to the estimation of the binomial parameter  $N$ . Biologists are interested in estimating the population sizes of different types of organisms in a plankton sample by making use of their count in the subsamples. A method is presented for determining subsample sizes for the different types of organisms in the

original sample. Also, the maximum likelihood estimators of the population sizes and their asymptotic properties are examined. The author also proposes a test for investigating the clustering of the organisms and the interval estimation problem of the organism count is examined for this case. Finally, the author demonstrates the use of the results obtained here using some plankton samples taken from the Chesapeake Bay.

This paper is accepted for publication and will appear in Biometrics in September, 1980.

In [3] above, a simple graphical method is described for obtaining the maximum likelihood of an integer-valued parameter. The method does not use calculus and is easy to comprehend. The use of this method is shown in the specific cases of the binomial, Poisson, and the exponential distributions. A numerical example is also provided for the comparison of this method with the method used previously.

In [4] above, Dahiya considers the estimation of the sample size  $N$  from truncated samples and the use of the Pearson chi-square goodness-of-fit test statistic for the situation. In order to use the Pearson chi-square goodness-of-fit test statistic for testing the adequacy of the assumed model, one needs to examine the effect of the estimation of  $N$  on the distribution of this test statistic. Here, it is shown that, when  $N$  itself is estimated, the asymptotic distribution of this test statistic is the same as the weighted sum of two independent chi-square random variables. Furthermore, the asymptotic distribution of this statistic for specific models is considered. An example is given to illustrate the use of the results obtained here.

### References

- Campbell, G. (1979). Nonparametric bivariate estimation with randomly censored data. Purdue University, Department of Statistics, Mimeo Series #79-25.
- Campbell, G., and Földes, A. (1980). Large sample properties of nonparametric bivariate estimators with censored data. Purdue University, Department of Statistics, Mimeo Series #80-10.
- Doksum, K. (1974). Empirical probability plots and statistical inference for nonlinear models in the two-sample case. Ann. Statist. 2, 267-77.
- Ferguson, T.S. (1973). A Bayesian analysis of some nonparametric problems. Ann. Statist. 1, 209-30.
- Kaplan, E.L., and Meier, P. (1958). Nonparametric estimation from incomplete observations. J. Amer. Statist. Assoc. 53, 457-81.
- Korwar, R.M., and Hollander, M. (1976). Empirical Bayes estimation of a distribution function. Ann. Statist. 4, 581-8.
- Korwar, R.M., and Dahiya, R.C. (1979). Estimation of a bivariate distribution function from incomplete observation. University of Massachusetts, Department of Mathematics and Statistics, AFOSR Tech Report #2.



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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER <b>AFOSR-TR- 80 - 0948</b>	2. GOVT ACCESSION NO. <b>AD-A090330</b>	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle)  <b>NONPARAMETRIC TESTS OF INDEPENDENCE FOR CENSORED DATA</b>		5. TYPE OF REPORT & PERIOD COVERED  <b>Final</b>
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s)  <b>Ramesh M. Korwar Ram C. Dahiya</b>		8. CONTRACT OR GRANT NUMBER(s)  <b>F49620-79-C-0105</b>
9. PERFORMING ORGANIZATION NAME AND ADDRESS <b>University of Massachusetts Department of Mathematics &amp; Statistics Amherst, MA 01003</b>		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS  <b>61102F 2304/A5</b>
11. CONTROLLING OFFICE NAME AND ADDRESS <b>Air Force Office of Scientific Research/NM Bolling AFB, Washington, DC 20332</b>		12. REPORT DATE <b>July 1980</b>
		13. NUMBER OF PAGES <b>Six</b>
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report)  <b>UNCLASSIFIED</b>
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  <b>Approved for public release; distribution unlimited.</b>		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) <b>Non parametric estimation                      self-consistent estimator Doubly censored data                              asymptotic properties Bivariate survivorship function</b>		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) <b>The work accomplished is represented by six Tech Reports already issued. Papers based on two of them are accepted for publication and are soon to be published in two of the leading journals in statistics. One other is submitted for publication. And yet another will appear in a Proceedings of a conference on nonparametric statistics.</b>		